ST2-Tutorübung – Lösung zu Blatt 3

1. Umladen von Kapazitäten

a) $i_{C1}=C_1\cdot\dot{u}_{C1}$

b) $i_{C1}=-i_R=-\frac{u_R}{R}=-\frac{(u_{C1}-u_{C2})}{R}=-\frac{u_{C1}}{R}+\frac{u_{C2}}{R}$

$i_{C2}=i_R=-i_{C1}=\frac{u_{C1}}{R}-\frac{u_{C2}}{R}$

c) $\dot{u}_{C1}=\frac{i_{C1}}{C_1}=-\frac{u_{C1}}{RC_1}+\frac{u_{C2}}{RC_1}$

$\dot{u}_{C2}=\frac{i_{C2}}{C_2}=\frac{u_{C1}}{RC_2}-\frac{u_{C2}}{RC_2}$

$$\begin{pmatrix}\dot{u}_{C1}\\ \dot{u}_{C2}\end{pmatrix}=\begin{pmatrix}-\frac{1}{RC_1} & \frac{1}{RC_1}\\ \frac{1}{RC_2} & -\frac{1}{RC_2}\end{pmatrix}\begin{pmatrix}u_{C1}\\ u_{C2}\end{pmatrix}$$

d) $\lambda_{1,2}=\frac{sp(\boldsymbol{A})}{2}\pm\sqrt{\left(\frac{sp(\boldsymbol{A})}{2}\right)^2-det(\boldsymbol{A})}=-\frac{1}{RC}\pm\sqrt{\left(\frac{1}{RC}\right)^2-0}=-\frac{1}{RC}\pm\frac{1}{RC}=\begin{cases}0\\ -\frac{2}{RC}\end{cases}$

weil $det(\boldsymbol{A})=\left(\left(\frac{1}{RC}\right)^2-\left(\frac{1}{RC}\right)^2\right)=0$

$\boldsymbol{q_1}\sim\begin{pmatrix}-a_{12}\\ a_{11}-\lambda_1\end{pmatrix}\sim\begin{pmatrix}-\frac{1}{RC}\\ -\frac{1}{RC}-0\end{pmatrix}\sim\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $\qquad$ $\boldsymbol{q_2}\sim\begin{pmatrix}-a_{12}\\ a_{11}-\lambda_2\end{pmatrix}\sim\begin{pmatrix}-\frac{1}{RC}\\ -\frac{1}{RC}-\left(-\frac{2}{RC}\right)\end{pmatrix}\sim\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

e) Alle Eigenwerte haben Realteil ≤ 0 $\Rightarrow$ stabil (aber nicht asymptotisch stabil, weil $\lambda_1=0$).
Oder: stabil, da nur aus passiven Elementen bestehend.

f) $\boldsymbol{Q}^{-1}=\boldsymbol{Q}^{\boldsymbol{H}}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ (weil $\boldsymbol{Q}$ unitär ist)

$$\boldsymbol{\Lambda}=\boldsymbol{Q}^{-1}\boldsymbol{A}\boldsymbol{Q}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}-\frac{1}{RC} & \frac{1}{RC}\\ \frac{1}{RC} & -\frac{1}{RC}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}=\frac{1}{2}\begin{pmatrix}0 & 0\\ -\frac{2}{RC} & \frac{2}{RC}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & -\frac{2}{RC}\end{pmatrix}$$

g) $\dot{\xi}=\boldsymbol{\Lambda}\xi=\begin{pmatrix}0 & 0\\ 0 & -\frac{2}{RC}\end{pmatrix}\xi \;\Rightarrow\; \xi=\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}=\begin{pmatrix}\xi_{01}\cdot e^{0\cdot t}\\ \xi_{02}\cdot e^{-\frac{2t}{RC}}\end{pmatrix}=\begin{pmatrix}\xi_{01}\\ \xi_{02}\cdot e^{-\frac{2t}{RC}}\end{pmatrix}$

h) $\xi=\boldsymbol{Q}^{-1}\boldsymbol{x} \;\Rightarrow\; \begin{pmatrix}\xi_{01}\\ \xi_{02}\end{pmatrix}=\boldsymbol{\xi_0}=\boldsymbol{Q}^{-1}\boldsymbol{x_0}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}U_0\\ 0\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}U_0$

i) $\boldsymbol{x}=\boldsymbol{Q}\boldsymbol{\xi} \Rightarrow u_{C1}(t)=x_1(t)=\frac{1}{\sqrt{2}}(\xi_1(t)+\xi_2(t))=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}U_0+\frac{1}{\sqrt{2}}U_0 e^{-\frac{2t}{RC}}\right)=\frac{U_0}{2}\left(1+e^{-\frac{2t}{RC}}\right)$

j) $u_{C1,\infty}=u_{C2,\infty}=\frac{U_0}{2}$

k) $i_R=i_{C2}=C_2\cdot\dot{u}_{C2}=C\cdot\dot{u}_{C2}$

l) $i_R(t)=C\,\dot{u}_{C2}(t)=C\frac{U_0}{2}\left(-e^{-\frac{2t}{RC}}\right)\left(-\frac{2}{RC}\right)=\frac{U_0}{R}e^{-\frac{2t}{RC}}$

$p_R(t)=R\,i_R^2=\frac{U_0^2}{R}e^{-\frac{4t}{RC}}$

m) $\Delta E=\int_0^\infty p(t)dt=\frac{U_0^2}{R}\left[e^{-\frac{4t}{RC}}\left(-\frac{RC}{4}\right)\right]_0^\infty=\frac{U_0^2}{R}\frac{RC}{4}=\frac{1}{4}C\,U_0^2$

n) Die Energie bleibt unverändert, da sie nicht von R abhängt.

o) $E(0)=E_{C1,0}+E_{C2,0}=\frac{1}{2}C\,U_0^2+0=\frac{1}{2}C\,U_0^2$

$E(\infty)=E_{C1,\infty}+E_{C2,\infty}=\frac{1}{2}C\left(\frac{U_0}{2}\right)^2+\frac{1}{2}C\left(\frac{U_0}{2}\right)^2=\frac{2\cdot 1}{2}C\frac{U_0^2}{4}=\frac{1}{4}C\,U_0^2$

$E(0)-E(\infty)=\Delta E$ (ok)